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Quasi-Diffusion Between Phonon and Roton Gases in Two- and Three-Dimensional Liquid Helium

by

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Quasi-diffusion between phonon and roton gases in two- and three-dimensional liquid helium

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Temperature variations of the quasi-diffusion constant and thermal diffusion ratio between phonon and roton gases in two- and three-dimensional liquid helium are evaluated explicitly through the scattering of phonons by rotons. The diffusion constants in both dimensions decrease exponentially over the whole temperature range, while the thermal diffusion ratios increase and then decrease as temperature increases in the range from ~ 0.6 to ~ 1.1 K.

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1. Introduction

Since Landau's phenomenological theory of the superfluids,[1] it is well known that liquid helium at low frequencies, long wavelengths and low temperatures can be described by Landau's two-fluid equations.[2] These equations involve not only the normal and superfluid density and thermodynamic functions, but also the various sounds and transport kinetic coefficients in liquid ^{3}He and ^{4}He. Concerning the kinetic coefficients, Landau and Khalatnikov[3] first investigated the thermal conductivity and viscosity, which were also studied later by others.[4] Diffusive processes[5] in liquid ^{3}He and ^{3}He-^{4}He mixtures have been extensively studied: Reuppeiner et al[6] measured the thermal conductivity (K), thermal diffusion ratio (K_T) and mass diffusivity (D) as a function of ^{3}He number concentration, and Behringer and Meyer[7] investigated diffusive relaxation processes in the normal and superfluid phase in liquid ^{3}He-^{4}He mixtures. All these quantities mentioned above are closely dependent on the elementary excitation, which is appropriate to describe the given Bose system. Motivated by these studies of ^{3}He-^{4}He mixtures, in the present paper we evaluate the temperature dependence of the diffusion constant (D) and thermal diffusive ration (K_T) between the phonon and roton gases in consideration of the scattering of phonons by rotons in two- and three-dimensional liquid ^{4}He.

Recently we have obtained microscopically the Landau-type excitation spectrum, which is (anomalous) phonon-like at low momentum and roton-like for large momenta in the ring diagram approximation.[8] Using this excitation spectrum we have successfully analyzed the sounds,[9] sound attenuations,[10] kinetic coefficients,[11] etc., in two- and three-dimensional liquid ^{4}He.

Through the calculations we can consider the excitation spectra in both dimensions:

$$\varepsilon(p) = C_0 P[1 + \gamma P^2 - \delta P^4 + \ldots] \quad , \qquad (2D) \qquad (1.1)$$

$$\varepsilon(p) = C_0 P[1 + \delta_1 P^2 - \delta_2 P^4 + \ldots] \quad , \qquad (3D) \qquad (1.2)$$

and

$$\varepsilon(p) = \Delta + \frac{(P-P_0)^2}{2\mu} \quad , \qquad (2D \text{ and } 3D) \qquad (1.3)$$

where P is the roton momentum, C_0 is the sound velocity, and Δ, P_0 and μ are the roton parameters (energy gap, momentum and effective mass, respectively). The subscript zero refers to T = 0 K. All coefficients in Eqs. (1.1) and (1.2) are positive, which are determined by the potential parameters. Here, we have adopted a soft potential with a Lennard-Jones type tail, which helps to make a smooth connection of the attractive part with the soft repulsive core. In the next section we evaluate the differential phonon-roton scattering cross section, and calculations of the diffusion constant and thermal diffusion ratio are described in Sec. 3. Finally, the results and discussion are presented in Sec. 4.

2. Phonon-roton differential scattering cross section

To obtain the two-dimensional phonon-roton scattering cross section, in this section we consider a roton in the presence of the phonon field. We can treat this roton as a particle in a moving liquid ^{4}He. Then there appears an additional term $-\vec{P}\cdot\vec{v}$, which can be written in symmetric form[12]

$$-\frac{1}{2}(\vec{P}\cdot\vec{v} + \vec{v}\cdot\vec{P}) \quad , \qquad (2.1)$$

where $\vec{P}$ and $\vec{v}$ are the momentum and velocity operators.[9] The phonon field changes the density of the medium, and thus we may expand the roton energy [Eq. (1.3)] in terms of the density ρ' to second order ($\rho' = \rho - \rho_0$) as

$$H_r = H_{ro} + \frac{\partial}{\partial \rho}\left(\Delta + \frac{1}{2\mu}(P-P_0)^2\right)\rho' + \frac{1}{2!}\frac{\partial^2}{\partial \rho^2}\left(\Delta + \frac{1}{2\mu}(P-P_0)^2\right)\rho'^2 + \ldots \quad , \tag{2.2}$$

where H_{ro} is given by Eq. (1.3). Since the value of most rotons is close to P_0, we may neglect $(P-P_0)$ and replace P by P_0. We may also drop the term $(\frac{\partial \Delta}{\partial \rho}\rho')$, which is much smaller than (2.1). Then the interaction energy between phonons and rotons can be written as

$$V = -\frac{1}{2}(\vec{P}\cdot\vec{v} + \vec{v}\cdot\vec{P}) + \frac{1}{2}\left(\frac{\partial^2 \Delta}{\partial \rho^2} + \frac{1}{\mu}\left(\frac{\partial P_0}{\partial \rho}\right)^2\right)\rho' \quad , \tag{2.3}$$

where the terms in the second bracket of Eq. (2.3) have magnitudes on the order of 10^{-1} to 1 in three-dimensional liquid helium.[13]

When the roton changes momentum $\vec{P}$ to $\vec{P}'$, it absorbs a phonon with momentum $\vec{p}$ and emits a phonon with momentum $\vec{p}'$. Here we may consider two intermediate processes: (I) $\vec{P} + \vec{p} \rightarrow \vec{P}' = \vec{P} + \vec{p} - \vec{p}'$ and (II) $\vec{P} - \vec{p}' \rightarrow \vec{P}' = \vec{P} - \vec{p}' + \vec{p}$. Since the roton momentum is much larger than that of the phonon, we may view this interaction as similar to that of heavy and light particles. The momentum and energy conservation law in collision processes can be written as

$$cp + \frac{1}{2\mu}(P-P_0)^2 = cp' + \frac{1}{2\mu}(|\vec{P} + \vec{p} - \vec{p}'| - \vec{P}_0)^2 \quad . \tag{2.4}$$

Under the conditions $p,p' \ll P_0$ and $E = cp \ll 3\mu c^2$, Eq. (2.4) becomes

$$p - p' = \frac{1}{2\mu c P_0^2} \{P_0 \hat{m}\cdot(p\hat{n} - p'\hat{n}')\}^2$$

$$= \frac{p^2}{2\mu C} \{\hat{m}\cdot(\hat{n}-\hat{n}')\}^2 \quad , \tag{2.5}$$

where $\hat{m}$, $\hat{n}$ and $\hat{n}'$ are the unit vectors directed along $\vec{P}_0$, $\vec{p}$ and $\vec{p}'$, respectively. Therefore energy conservation implies $p \simeq p'$. This means that particles do not change the magnitude of momentum, but change its direction. Taking account of $p,p' \ll P_0$ and $P = P_0$, the matrix element given in the differential cross section

$$d\sigma = (\frac{2\pi}{\hbar c_0}) |\langle F|H_3|I\rangle|^2 \, \delta(E_F - E_I) \, \frac{d\vec{P}}{(2\pi\hbar)^2} \tag{2.6}$$

can be written in second-order perturbation as

$$\langle F|H_3|I\rangle = \frac{P_0 P}{2\rho_0} \{[(\hat{m}\cdot\hat{n}) + (\hat{m}\cdot\hat{n}')] \, (\hat{n}\cdot\hat{n}') + \frac{P_0}{\mu c}(\hat{m}\cdot\hat{n})^2(\hat{m}\cdot\hat{n}')^2 + A\} \quad , \tag{2.7}$$

$$A = \frac{\rho^2}{P_0 c} \{\frac{\partial^2 \Delta}{\partial \rho^2} + \frac{1}{\mu}(\frac{\partial P_0}{\partial \rho})^2\} \quad . \tag{2.8}$$

Substituting Eqs. (2-7)-(2.8) into Eq. (2.6) and performing the integration over p', we obtain

$$d\sigma = \frac{P_0^2 p^3}{8\pi\hbar^3 \rho_0^2 c^2} \{[(\hat{m}\cdot\hat{n}) + (\hat{m}\cdot\hat{n}')](\hat{m}\cdot\hat{n}') + \frac{P_0^2}{\mu c}(\hat{m}\cdot\hat{n})^2(\hat{m}\cdot\hat{n}')^2 + A^2\} d\theta \quad . \quad (2.9)$$

Averaging Eq. (2.9) over all directions of roton momentum, we finally obtain

$$d\sigma = \frac{P_0^2 p^3}{8\pi\hbar^3 \rho_0^2 c^2} \{(1 + \cos\psi)\cos^2\psi + \frac{1}{128}(\frac{P_0}{\mu c})^2 (35\cos^4\psi + 3\sin^4\psi$$

$$+ 30\cos^2\psi \sin^2\psi) + \frac{1}{4}\frac{P_0 A}{\mu c}(3\cos^2\psi + \sin^2\psi) + A^2\} d\psi \quad ,$$

where ψ is the angle between the incident and scattered phonons.

Through the similar calculations we can obtain the differential scattering cross section of phonons by rotons in bulk liquid ^{4}He:[14]

$$d\sigma = (\frac{P_0 p}{4\pi\hbar^2 \rho_0 c})^2 \{\frac{2}{3}(1 + \cos\psi)\cos^2\psi + \frac{1}{105}(\frac{P_0}{\mu c})^2(1 + 8\cos^2\psi + \frac{3}{8}\cos\psi)$$

$$+ \frac{2A}{12}(\frac{P_0}{\mu c})(1 + 2\cos^2\psi) + A^2\} d\Omega \quad , \quad (2.11)$$

where $d\Omega$ is the solid angle.

3. Diffusion constant and thermal diffusion ratio

Since the excitation spectrum in liquid ^{4}He consists of phonons and rotons, we may treat liquid ^{4}He as a mixed phonon-roton gas. In the temperature range from ~0.7 to ~1.2 K, where the scattering of phonons by rotons is dominant, we consider the diffusion of phonons and rotons and also

the thermal diffusion ratio under the existence of a temperature gradient. There thermal processes in the mixture of a phonon-roton gas are very similar to those of light particles in heavy particles. Since the roton momentum is much larger than that of the phonon, we may treat the interactions between phonons and rotons as similar to those between light and heavy particles. Therefore, we may assume that when a collision occurs only between a phonon and roton, the roton is at rest and the phonon momentum changes only its direction with the same magnitude.

Let n_p and n_r be the number densities of the phonon and roton, respectively, and θ the angle between the direction of a phonon with momentum $\vec{p}$ and the x-axis. Equation (2.11) represent the differential scattering cross section that the momentum $\vec{p}$ of a phonon will be changed to the momentum $\vec{p}'$ directed into the solid angle element $d\Omega$. The collision probability of a phonon per unit time becomes $n_r c d\sigma$, where c is the phonon velocity. Let the number of phonons in a given unit volume with momentum between p and p + dp and directed into the solid angle $d\Omega$ be $n(p,\theta,x)d^3p$. Then the total number of phonons changing $\vec{p}$ to $\vec{p}'$ in $d\Omega$ and conversely $\vec{p}'$ to $\vec{p}$ in $d\Omega'$ are, respectively, given by

$$d^3p \int n_r c n(p,\theta,x)\, d\sigma \quad , \tag{3.1}$$

$$d^3p \int n_r c n(p',\theta',x)\, d\sigma \quad . \tag{3.2}$$

The difference between the above two processes,

$$d^3p\, n_r c \int [n(p',\theta',x) - n(p,\theta,x)]\, d\sigma \quad , \tag{3.3}$$

represents the change in the number of phonons in the volume element d^3p, which must be equal to the total time derivative of n:

$$J_{phr}(n) = c\cos\theta\frac{\partial n}{\partial x} = n_r c\int[n(c',\theta',x) - n(p,\theta,x)]\,d\sigma' \quad . \tag{3.4}$$

If the temperature and concentration gradients vary very slowly, we can express the phonon distribution function as

$$n = n_0(p,x) + \delta n(p,\theta,x) \quad , \tag{3.5}$$

where δn is a small correction term to the equilibrium distribution function n_0. Now we assume that δn is linear in the gradient of the concentration and temperature, and has the form

$$\delta n = \cos\theta f(p,x) \quad . \tag{3.6}$$

Substitution of Eq. (3.6) and $\cos\theta' = \cos\theta\cos\psi + \sin\theta\sin\psi\cos(\psi-\psi')$ in Eq. (3.4) and then integration over the solid angle yields

$$J_{phr}(n) = \cos\theta\; n_r c f(p,x)\,\frac{a^4 p_0^2 k_B^4 T^4}{4\pi\hbar^4\rho_0^2 c^6}\;\Gamma \quad , \tag{3.7}$$

$$\Gamma = \frac{4}{45} + \frac{1}{25}\left(\frac{P_0}{\mu c}\right)^2 + \frac{2}{9}\left(\frac{P_0 A}{\mu c}\right) + A_0^2 \quad .$$

Here we have made use of the relation $p = ak_B T/c$, where a is a constant[15] (see the Appendix). From Eqs. (3.4) and (3.7), we obtain the following expression for f(p,x):

$$f(p,x) = -\frac{1}{n_r}\frac{4\pi\hbar^4\rho_0^2c^6}{a^4P_0^2\Gamma k_B^4T^4}\frac{\partial n}{\partial x} \quad . \tag{3.8}$$

The diffusion flux i along the x-axis is

$$i = \int nc\cos\theta\, d^3p = c\int \cos^2\theta\, f(p,x)d^3p \quad . \tag{3.9}$$

Substituting Eq. (3.8) into Eq. (3.9) and integrating over momentum p gives

$$i = \frac{4\pi\hbar^4\rho_0^2c^7}{3a^2P_0^2\Gamma n_r k_B^4T^4}\frac{\partial n_p}{\partial x} \quad . \tag{3.10}$$

Let the phonon concentration of the mixture be $\eta = n_p/(n_0+n_r) = n_0/n$. Then the diffusion flux i becomes

$$i = \frac{4\pi\hbar^4\rho_0^2c^7}{3a^4P_0^2\Gamma n_r k_B^4T^4}\, n[\frac{\partial \eta}{\partial x} + \eta\frac{\partial}{\partial T}\ln(n)\frac{\partial T}{\partial x}] \quad . \tag{3.11}$$

Comparing Eq. (3.11) with the expression[16] for the diffusion flux

$$i = -nD(\vec{\nabla}\eta + \frac{K_T}{T}\vec{\nabla}T) \quad , \tag{3.12}$$

we obtain

$$D = \frac{4\pi\hbar\rho_0^2 c^7}{3a^4 P_0^2 \Gamma n_r k_B^4} \quad , \tag{3.13}$$

$$K_T = \eta T \frac{\partial}{\partial T} \ln(n) \quad . \tag{3.14}$$

Since the number densities of phonons and rotons per unit volume can be given by

$$n_p = 2!\zeta(3)\ 4\pi\left(\frac{k_B T}{2\pi\hbar c}\right)^3 \quad , \qquad (3D) \tag{3.15}$$

$$n_r = \frac{2P_0^2(\mu k_B)^{1/2}}{(2\pi\hbar^2)^{3/2}} T^{1/2}\ e^{-\Delta/k_B T} \quad , \qquad (3D) \tag{3.16}$$

we obtain the diffusion coefficient and thermal diffusion ratio of phonons in bulk liquid helium:

$$D(T) = \frac{(2\pi)^{5/2}\rho_0^2(\hbar c)^7}{3a^4 P_0^4 \mu^{1/2}\Gamma k_B^{9/2}} T^{-9/2}\ e^{\Delta/k_B T} \quad , \tag{3.17}$$

$$K_T = \frac{3 + \left(\frac{\zeta(3)}{\sqrt{2}}\right)^{-1}(\pi\mu)^{1/2}\ P_0^2 c^3 \left(\frac{1}{2} + \frac{\Delta}{k_B T}\right)(k_B T)^{-5/2}\ e^{-\Delta/k_B T}}{\left[1 + \left(\frac{\zeta(3)}{\sqrt{2}}\right)^{-1}(\pi\mu)^{1/2}\ P_0^2 c^3 (k_B T)^{-5/2}\ e^{-\Delta/k_B T}\right]^2} \quad . \tag{3.18}$$

Through similar calculations as done above, we obtain the diffusion coefficient and thermal diffusion ratio in thin helium films as

$$D(T) = \frac{(2^5\pi)^{1/2}\rho_0^2\hbar^5c^6}{a'^{13}P_0^3\mu^{1/2}\Gamma k_B^{7/2}}\, T^{-7/2}\, e^{\Delta/k_BT} \quad , \tag{3.19}$$

$$K(T) = \frac{2 + (\zeta(2))^{-1}(2\pi\mu)^{1/2}P_0c^2(\frac{1}{2} + \frac{\Delta}{k_BT})(k_BT)^{-3/2}e^{-\Delta/k_BT}}{[1 + (\zeta(2))^{-1}\,(2\pi\mu)^{1/2}P_0c^2(k_BT)^{-3/2}e^{-\Delta/K_BT}]^2} \quad , \tag{3.20}$$

where we used the number densities of phonons and rotons per unit surface given by

$$n_p = \frac{\zeta(2)}{2\pi}\,(\frac{k_BT}{\hbar c})^2 \quad . \tag{3.21}$$

$$n_r = \frac{P_0}{2\pi\hbar^2}\,(2\pi\mu k_BT)^{1/2}\, e^{-\Delta/k_BT} \quad , \tag{3.22}$$

and Γ' is given as

$$\Gamma' = \frac{1}{4} + \frac{9}{32}\,(\frac{P_0}{\mu c})^2 + \frac{P_0A}{\mu c} + 2A^2 \quad .$$

4. Results and discussion

Since the elementary excitation spectrum in liquid helium consists of the combined phonon-roton gas, we may consider the diffusion of phonons into the roton gas, and also may include the thermal diffusion under the existence of the temperature gradient. We may treat the diffusion and thermal diffusion of the phonons in the roton gas as similar to those of light particles in heavy particle.. For convenience we neglected phonon-phonon or roton-roton scatterings, permitting only phonon-roton scatterings.

The thermal diffusion ratio represents the ratio of the thermal diffusion coefficient due to the temperature gradient to the diffusion coefficient given by the concentration gradient. The diffusion coefficient must be positive, but the diffusion ratio may be positive or negative. The negative value of the coefficient K_T means that the heavy particles will appear in the low-temperature region and the light particles in the high-temperature region. In ^{3}He-^{4}He mixtures[17] the ratio K_T becomes negative near the plait point and positive near transition temperature. Since thermal diffusion ratios in our calculations are positive, phonons and rotons are located in the low- and high-temperture regions, respectively.

The diffusion constant and thermal diffusion ratios of the phonon gas in bulk liquid helium and in thin helium films are given by Eqs. (3.17)-(3.20). The potential and roton parameters are chosen by the analysis of the excitation spectrum[18] for the bulk case and the specific data[8] for the thin helium films (Table I). For the density variations of the roton parameters, we have adopted the results obtained by Khalatnikov,[14] who analyzed the data of Dietrich et al[19] given by

$$\left(\frac{\rho}{\Delta}\right)\left(\frac{\partial\Delta}{\partial\rho}\right) = -1.142, \qquad \left(\frac{\rho^2}{\Delta}\right)\left(\frac{\partial^2\Delta}{\partial\rho^2}\right) = -2.06, \qquad \left(\frac{\rho}{P_0}\right)\left(\frac{\partial P_0}{\partial\rho}\right) = \frac{1}{3} \quad .$$

The above parameters in thin helium films are not known and thus are assumed to have the same values in the bulk case. The constants a and a′ in Eqs. (3.7) and (3.19) can be determined by the calculation of the collision time for the phonon-roton scatterings, which characterize the thermal conductivity and viscosity. The numerical values of these a and a′ are 6.28 and 4.67, respectively.[15]

Figure 1 illustrates the diffusion constant D as a function of temperature in the bulk and for films. As temperature increases, the roton density increases more rapidly than the phonon density. Therefore, the mean free path of phonons becomes shorter and the coefficient decreases rapidly.

The temperature variations of the coefficients K_T are shown in Fig. 2. Below ~ 0.6 K the coefficient is almost constant. In the temperature range from ~ 0.6 to ~ 1.1 K the coefficient ratio increases, which means that phonon-roton scatterings play a main role in the transport phenomena. This temperature range agrees well with the results obtained by Landau and Khalatnikov.[12] For temperature far below ~ 0.6 K, the phonon population is dominant, and thus phonon-phonon interactions becomes important, while roton-roton interactions and five-phonon processes will be dominant above ~ 1.1 K.

In thin helium films the temperature range in which phonon-roton scatterings are effective and dominant becomes ~ 0.3 K < T < 0.8 K. This range is in good agreement with the results obtained from the evaluation of kinetic coefficients in thin helium films.[9] All arguments metioned with respect to the bulk helium will be valid.

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Appendix

Making use of Eqs. (3.13)-(3.15), (3.21) and (A11) from Ref. 15, we can easily obtain the characteristic time $\tau^{-1}_{ph\text{-}r}$ expressed as a function of the phonon momentum and temperature, respectively, as

$$\tau^{-1}_{ph\text{-}r} = N_r \frac{P_0 p^3}{8\hbar^3 \rho^2 c} \left[\frac{1}{4} + \frac{9}{32}\left(\frac{P_0}{\mu c}\right)^2 + \frac{P_0}{\mu c} A + 2A^2\right] , \tag{A1}$$

and

$$\tau^{-1}_{ph\text{-}r} = \frac{6!\zeta(6)}{3!\zeta(3)} N_r \frac{P_0^2 k_B^3 T^3}{8\hbar^3 \rho^2 c^4} \left[\frac{1}{4} + \frac{9}{32}\left(\frac{P_0}{\mu c}\right)^2 + \frac{P_0}{\mu c} A + 2A^2\right] , \tag{A2}$$

where N_r is the number of rotons per unit area. Setting (A1) equal to (A2), we get

$$pc = \left[\frac{6!\zeta(6)}{3!\zeta(3)}\right]^{1/3} k_B T . \tag{A3}$$

Therefore, we obtain $a' = 4.67$ in two-dimensional liquid helium.

Through a similar calculation we can evaluate the characteristic time expressed by temperature and phonon momentum:

$$\tau^{-1}_{ph\text{-}r} = 4\pi c N_r \left(\frac{P_0 p^2}{4\pi\hbar^2 \rho c}\right)^2 \left[\frac{4}{45} + \frac{1}{25}\left(\frac{P_0}{\mu c}\right)^2 + \frac{2}{9}\left(\frac{P_0}{\mu c} A\right) + A^2\right] . \tag{A4}$$

and

$$\tau_{ph\text{-}r}^{-1} = \frac{4\pi}{c} N_r \left(\frac{P_0 k_B^2 T^2}{\hbar^2 \rho c^2}\right)^2 \left[\frac{4}{45} + \frac{1}{25}\left(\frac{P_0}{\mu c}\right)^2 + \frac{2}{9}\left(\frac{P_0 A}{\mu c}\right) + A^2\right] \quad . \tag{A5}$$

From Eqs. (A4)-(A5) we obtain $a = 2\pi$ in the bulk case.

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Table I. Theoretical Parameters

	η	Δ/k_B (K)	q_0 ($\AA^{-1}$)	μ	C (m/s)
3D	2.18×10^{-2} $\AA^{-3}$	8.616	1.930	$0.153m_{He}$	238.21
2D	2.79×10^{-2} $\AA^{-2}$	4.12	1.02	$0.77m_{He}$	164.4

Figure Captions

1. Diffusion constants of phonons in the roton gas as a function of temperature. The solid and dotted lines represent the theoretical curves in two- and three dimensional liquid ^{4}He.

2. Thermal diffusion ratio of phonons in the roton gas versus temperature. The solid and dotted lines represent the theoretical curves in two- and three-dimensional liquid ^{4}He.

Fig. 1

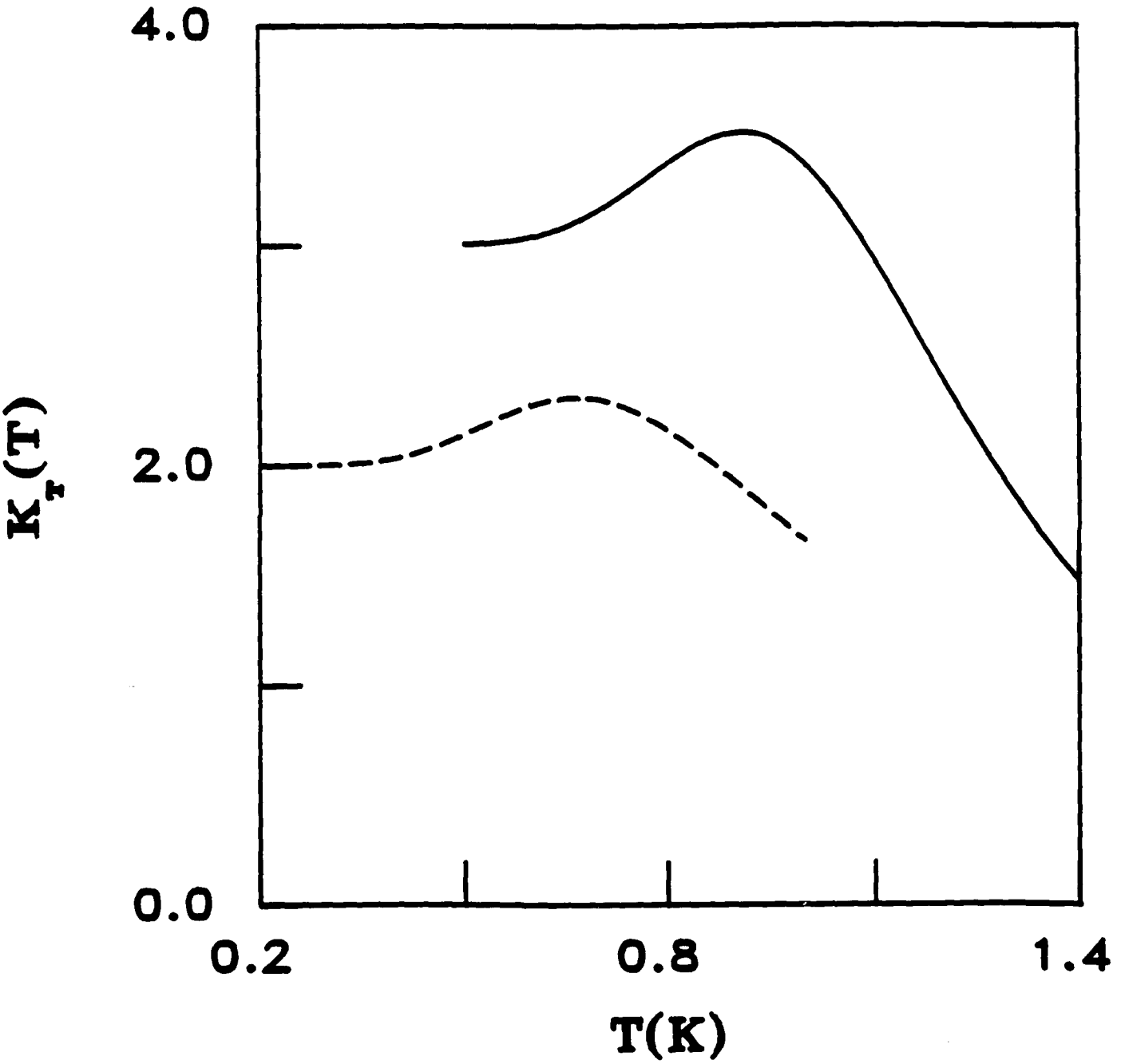
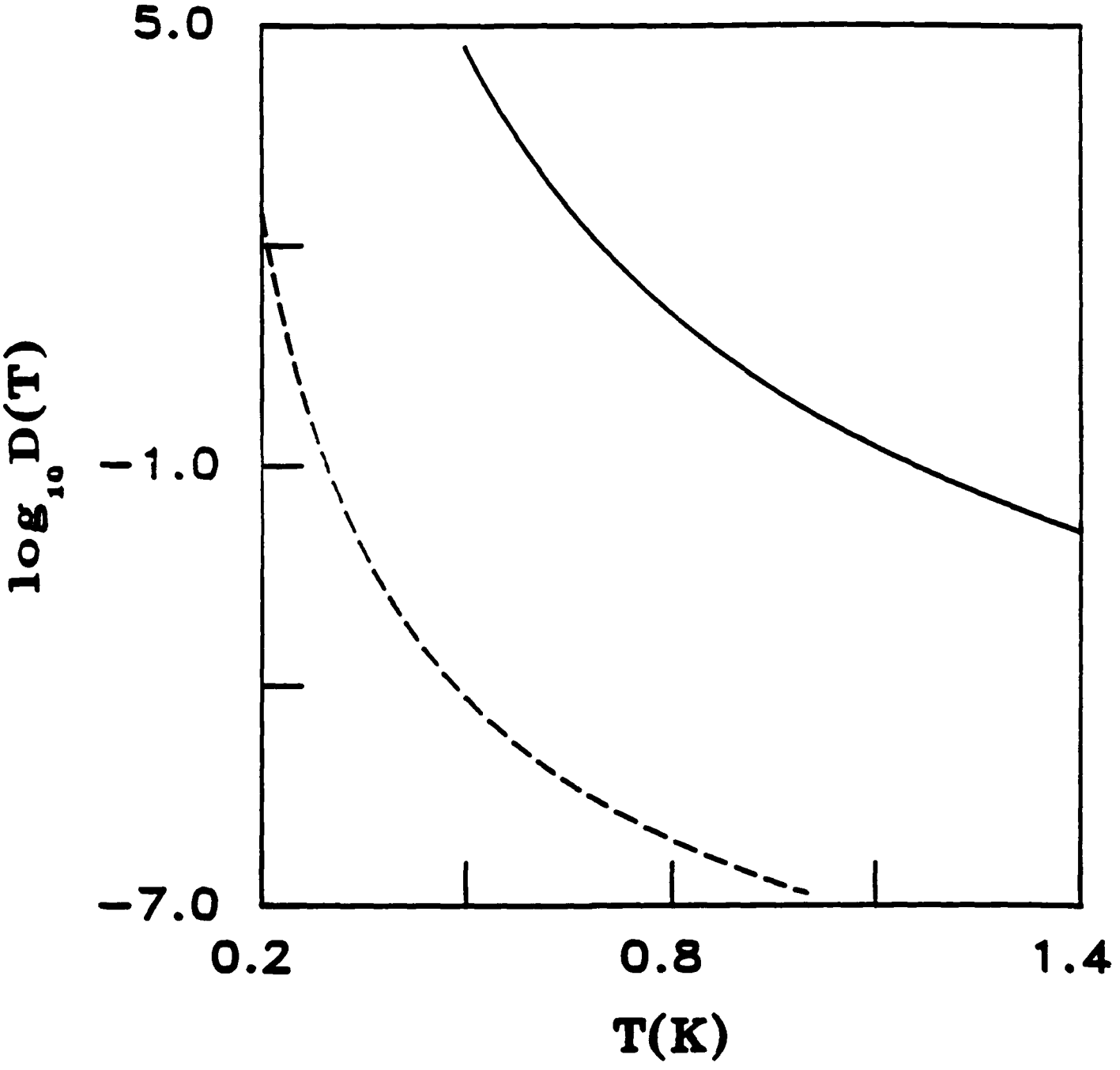


Fig. 2



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